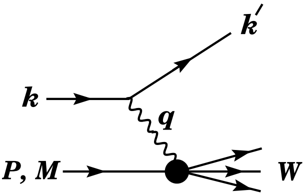
Lecture 10: The Structure of the Proton: Part II

Sept 27, 2016

Reminder: Deep Inelastic Scattering



- W is the invariant mass of the hadronic system
- In lab frame: $P = (M, 0)$
- In any frame, $k = k' + q$, $W = p + q$
- Invariants of the problem:

$$
\begin{aligned}
Q^2 &= -q^2 = -(k - k')^2 \\
&= 2EE'(1 - \cos\theta) \ \ \text{[in lab]} \\
P \cdot q &= P \cdot (k - k') \\
&= M(E - E') \ \ \text{[in lab]}
\end{aligned}
$$

- Define $\nu \equiv E - E'$ (in lab frame) so $P \cdot q = m\nu$ and

$$
\begin{aligned}
W^2 &= (P + q)^2 \\
&= (P - Q)^2 \\
&= M^2 + 2P \cdot q - Q^2 \\
&= M^2 + 2M\nu - Q^2
\end{aligned}
$$

where $Q^2 = -q^2$

- Elastic scattering corresponds to $W^2 = P^2 = M^2$
  - $Q^2 = 2M\nu$ elastic scattering
- We can define 2 indep dimensionless parameters

$$
\begin{aligned}
x &\equiv Q^2/2M\nu; \quad (0 < x \leq 1) \\
y &\equiv \frac{P \cdot q}{P \cdot k} = 1 - E'/E; \quad (0 < y \leq 1)
\end{aligned}
$$

Reminder: Structure Functions

- Using notation from previous page, we can express the x-section for DIS

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2}\frac{\cos^2(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)}\left[W_2(q^2,W) + 2W_1(q^2,W)\tan^2(\frac{1}{2}\theta)\right]$$

- W_1 and W_2 care called the *structure functions*
  - Angular dependence here comes from expressing covariant form on last page in lab frame variables
  - Two structure functions that each depend on Q^2 and W
  - Alternatively, can parameterize wrt dimensionless variables:

$$\begin{aligned} x &\equiv Q^2/2M\nu \\ y &\equiv \frac{P\cdot q}{P\cdot k} = 1 - E'/E \end{aligned}$$

Reminder: The Parton Model

- Supposed there are pointlike partons inside the nucleon
- Work in an "infinite momentum" frame: ignore mass effects
- Proton 4-momentum: $P = (P, 0, 0, P)$
- Visualize stream of parallel partons each with 4-momentum xP where $0 < x < 1$; neglect transverse motion of the partons
  - x is the fraction of the proton's momentum that the parton carries
- If electron elastically scatters from a parton

$$\begin{aligned} (xP + q)^2 = m^2 &\simeq 0 \\ x^2P^2 + 2xP \cdot q + q^2 &= 0 \end{aligned}$$

Since $P^2 = M^2$, if $x^2M^2 << q^2$ then

$$\begin{aligned} 2xP \cdot q &= -q^2 = Q^2 \\ x &= \frac{Q^2}{2P \cdot q} = \frac{q^2}{2M\nu} \end{aligned}$$

Deep inelastic scattering can be described as elastic scattering of the lepton with a parton with momentum xP

Writing the DIS cross section in terms of PDFs

- The cross section is incoherent sum over elastic scattering with partons
- If partons are quarks, they are Dirac particles and we can calculate everything:

$$\frac{d\sigma}{dE'd\Omega}|_{Dirac} = \frac{4\alpha^2 E'^2}{Q^2}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]\delta(\nu + q^2/2Mx)$$

- Taking incoherent sums:

$$\frac{d\sigma}{dE'd\Omega}|_{Dirac} = \sum_i \int_0^1 dx\ f_i(x)e_i^2\left[\cos^2\frac{\theta}{2} + Q^2/2M^2x^2\sin^2\frac{\theta}{2}\right]\delta(\nu + q^2/2Mx)$$

- Using the property of delta functions

$$\delta(g(x)) = \frac{\delta(x-x')}{|g'(x-x_0)|_{x=x_0}}$$

where $g(x_0) = 0$, we can write

$$\delta(\nu - Q^2/2Mx) = \frac{\delta(x - Q^2/2M\nu)}{Q^2/2Mx^2} = \frac{x}{v}\delta(x - Q^2/2M\nu)$$

- This gives us:

$$\frac{d\sigma}{dE'd\Omega} = \sum_i \int_0^1 dx\ f_i(x)e_i^2\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2x^2}\sin^2\frac{\theta}{2}\right]\frac{x}{\nu}\ \delta(x - Q^2/2M\nu)$$

Continuing from last page

- The result we just obtained on the previous page was

$$\frac{d\sigma}{dE'd\Omega} = \sum_i \int_0^1 dx\ f_i(x)e_i^2 \left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2x^2}\sin^2\frac{\theta}{2}\right]\frac{x}{\nu}\delta(x - Q^2/2M\nu)$$

- Notice that we have one term proportional to $\cos^2\frac{\theta}{2}$ and one proportional to $\sin^2\frac{\theta}{2}$
  - This is the same form as our phenomenological form for the ep scattering:

$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^2}{4E^2}\frac{1}{\sin^4(\frac{\theta}{2})}\left[W_2(x)\cos^2(\frac{\theta}{2}) + 2W_1(x)\sin^2(\frac{\theta}{2})\right]$$

  - Equating terms

$$\begin{aligned} W_2(x) &= \sum_i f_i(x)e_i^2\frac{x}{\nu} \\ W_1(x) &= \sum_i f_i(x)e_i^2\frac{Q^2}{2M\nu}\frac{x}{\nu}\delta(x - \frac{Q^2}{2M\nu}) \\ &= \sum_i f_i(x)\frac{e_i^2}{2M} \end{aligned}$$

- It's usual to define

$$F_2(x) \equiv \nu W_2(x) \qquad F_1(x) \equiv MW_1(x)$$

Then

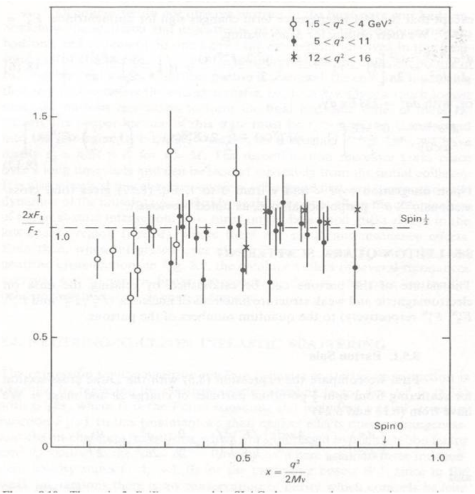
$$F_2(x) = \sum_i x\ f_i(x)e_i^2 \qquad F_1(x) = \textstyle\sum_i\ f_i(x)\frac{e_i^2}{2}$$

$$F_2(x) = 2xF_1(x)$$

This is called the Callen-Gross relation

- Note: If our partons had spin-0 rather than spin-$\frac{1}{2}$, we would have found $F_1 = 0$

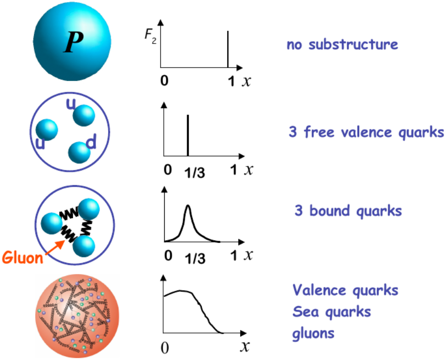
What does the data look like?



from Perkins

The partons act like spin-1/2 Dirac particles!

If partons are quarks, what do we expect?



Max Klein, CTEQ School Rhodos 2006

- Elastic scattering from proton has $x = 1$
- If 3 quarks carry all the proton's momentum each has $x = 0.3$
- Interactions among quarks smears $f(x)$
- Radiation of gluons softens distribution and adds $q\bar{q}$ pairs
  - Describe the 3 original quarks as "valence quarks"
  - $q\bar{q}$ pairs as sea or ocean
- Some of proton's momentum carried by gluons and not quarks or antiquarks

Using Isospin: Comparing the Proton and Neutron

- Ignore heavy quark content in the proton: consider only u, d, s
- Write the proton Structure Function

$$\frac{F_2^p(x)}{x} = \sum_i f_i^p(x)e_i^2 = \frac{4}{9}(u^p(x) + \overline{u}^p(x)) + \frac{1}{9}(d^p(x) + \overline{d}^p(x)) + \frac{1}{9}(s^p(x) + \overline{s}^p(x))$$

- Similarly, for the neutron

$$\frac{F_2^n(x)}{x} = \sum_i f_i^n(x)e_i^2 = \frac{4}{9}(u^n(x) + \overline{u}^n(x)) + \frac{1}{9}(d^n(x) + \overline{d}^n(x)) + \frac{1}{9}(s^n(x) + \overline{s}^n(x))$$

- But isospin invariance tells us that $u^p(x) = d^n(x)$ and $d^p(x) = u^n(x)$
- Write F_2 for the neutron in terms of the proton pdf's (assuming same strange content for the proton and neutron)

$$\frac{F_2^n(x)}{x} = \frac{4}{9}(d^p(x) + \overline{d}^p(x)) + \frac{1}{9}(u^p(x) + \overline{u}^p(x)) + \frac{1}{9}(s^p(x) + \overline{s}^p(x))$$

- Assuming sea q and $\overline{q}$ distributions are the same:

$$u(x) - \overline{u}(x) = u_v(x), \quad d(x) - \overline{d}(x) = d_v(x), \quad s(x) - \overline{s}(x) = 0$$

- Taking the difference in F_2 for protons and neutrons:

$$\frac{1}{x}[F_2^p(x) - F_2^n(x)] = \frac{1}{3}[u_v(x) - d_v(x)]$$

which gives us a feel for the valence quark distribution

What the data tells us

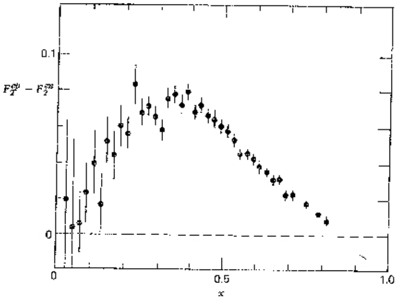
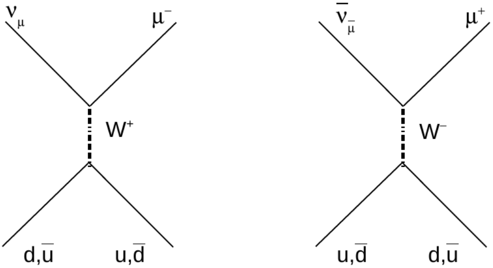


Fig. 9.8 The difference $F_2^{ep} - F_2^{en}$ as a function of x, as measured in deep inelastic scattering. Data are from the Stanford Linear Accelerator.

From Halzen and Martin

- Looks the way we expect from the cartoon on page 27
- Next question: How to measure the partons' charge
  - To do this, must compare e and ν scattering!

Neutrino-(anti)quark Charged Current Scattering (I)

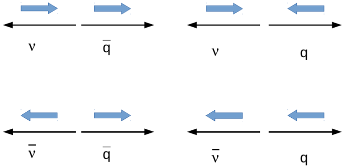


- Start with ν_μ or $\overline{\nu}_\mu$ beam
  - Ingoing ν 4-momentum correlated with radius of interaction
  - Outgoing $\mu^\pm$ momentum measured in spectrometer
- Exchange via $W^\pm$
  - ν scatter against d and $\overline{u}$
  - $\overline{\nu}$ scatter against u and $\overline{d}$

We'll talk about neutral currents in a few weeks
Not useful for structure function measurements
(Can't measure outgoing lepton 4-momentum)

Neutrino-(anti)quark Scattering (II)

- Neutrinos left handed, anti-neutrinos right handed
- Left handed $W^{\pm}$ couples to left-handed quarks and right-handed anti-quarks



- νq and $\overline{\nu q}$ scattering allowed for all angles, but $\overline{\nu} q$ and $\nu \overline{q}$ vanish in backward direction

$$\frac{d\sigma^{\nu q}}{d\cos\theta} \propto \text{constant} \qquad \frac{d\sigma^{\overline{\nu} q}}{d\cos\theta} = (1+\cos\theta^*)^2$$

where θ^* is scattering angle in νq center of mass

- We'll see later that this left-handed coupling is also reason that π and K preferentially decay to μ and not e
  - μ^- needs to be right-handed since π,K have spin 0
  - rh component of spinor $\propto (v/c) \propto m_\mu$ in matrix element; decay rate $\Gamma \propto m_\ell^2$

    This is why accelerators produce predominantly ν_μ,$\overline{\nu}_\mu$

Neutrino-(anti)quark Scattering (III)

- The charged current cross sections are v_μ:

$$\begin{aligned}
\frac{d\sigma(\nu_\mu\ d \to \mu^- u)}{d\Omega} &= \frac{G^2 s}{4\pi^2} \\
\frac{d\sigma(\overline{\nu_\mu}\ u \to \mu^+ d)}{d\Omega} &= \frac{G^2 s}{4\pi^2}\frac{(1+\cos\theta)^2}{4} \\
\frac{d\sigma(\nu_\mu\ \overline{u} \to \mu^- \overline{d})}{d\Omega} &= \frac{G^2 s}{4\pi^2}\frac{(1+\cos\theta)^2}{4} \\
\frac{d\sigma(\overline{\nu_\mu}\ \overline{d} \to \mu^+ \overline{u})}{d\Omega} &= \frac{G^2 s}{4\pi^2}
\end{aligned}$$

- You will prove on homework #5 that

$$1 - y = \frac{p \cdot k'}{p \cdot k} = \frac{1}{2}\left(1 + \cos\theta^*\right)$$

which allows us to rewrite the above expressions in terms of the relativistically invarient variable y

- When we integrate over $d\Omega$ the cross sections with the $(1+\cos\theta)^2$ factor are $\frac{1}{3}$ of the cross sections without that factor
- This means

$$\begin{array}{cccc}
\sigma^{\nu d} : & \sigma^{\nu \overline{u}} : & \sigma^{\overline{\nu} u} : & \sigma^{\overline{\nu}\overline{d}} \quad = \\
1 : & \frac{1}{3} : & \frac{1}{3} : & 1
\end{array}$$

What is the Advantage of ν Scattering?

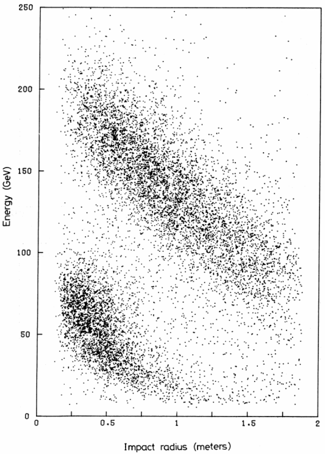
- The quarks and antiquarks have different angular dependence, so we can extract their pdf's separately by looking at cross sections as a function of angle
  - This angular dependence expressed as in terms of dimensionless variable y
  - Parity violation means we have a third structure function F_3 that I won't talk about today
- Weak "charge" of the u and d is the same, so factors of $4/9$ and $1/9$ are not present
- Using previous expressions and integrating over angle:

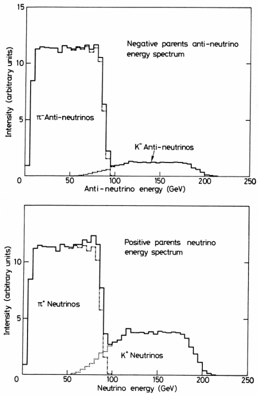
$$
\begin{aligned}
\frac{d\sigma(\nu\ p)}{dx} &= \frac{G^2 x s}{\pi}[d(x) + \frac{1}{3}\overline{u}(x)] \\
\frac{d\sigma(\nu\ n)}{dx} &= \frac{G^2 x s}{\pi}[d^n(x) + \frac{1}{3}\overline{u}^n(x)] \\
&= \frac{G^2 x s}{\pi}[u(x) + \frac{1}{3}\overline{d}(x)]
\end{aligned}
$$

where we have written everything in terms of the proton PDFs

- If we believe the partons in the proton and neutron are quarks, we can relate the structure functions measured in νN and eN

An Aside: How do we know the incoming neutrino energy?





- Primary proton beam incident on target produces secondary π and K
- Use magnets and shielding to select range of momenta of secondaries
- Long decay region to allow the $\pi \to \mu\nu$ and $K \to \mu\nu$ decays
- Two body decay gives correlation between decay angle and neutrino momentum

Comparing eN and νN νN Scattering (I)

- Now, let's take an isoscalar target N (equal number of protons and neutrons)
- In analogy with electron scattering

$$\frac{F_2^{\nu N}}{x} = u(x) + d(x) + \overline{u}(x) + \overline{d}(x)$$

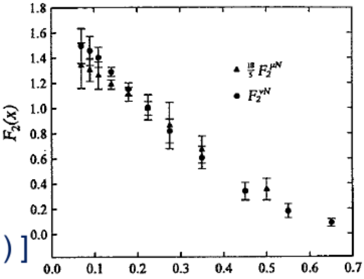
- If we go back to our electron scattering and also require an isoscalar target

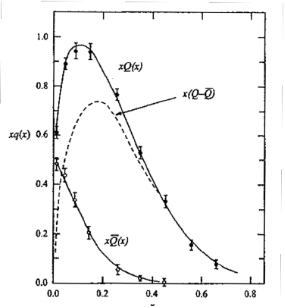
$$\frac{F_2^{e\ N}}{x} = \frac{5}{18}(u(x) + d(x) + \overline{u}(x) + \overline{d}(x))$$

- So, if the partons have the charges we expect from the quark model

$$F_2^{e\ N}(x) = \frac{5}{18} F_2^{\nu N}(x)$$

Comparing eN and νN νN Scattering (II)





- The partons we "see" in eN scattering are the same as the ones we "see" in νN scattering
- This confirms our assignment of the quark charges:

  The Quarks Have Fractional Charge!

Using νN scattering to Count Quarks and Antiquarks

- As we previously did for electron scattering, we can look at an isoscalar target N
- Starting with the cross sections for νq scattering we can go through the same convolution with the PDFs that we did for the eN case
- The result is

$$\sigma^{\nu N} = \frac{G2ME}{2\pi}\left[Q + \frac{1}{3}\overline{Q}\right]$$
$$\sigma^{\overline{n}uN} = \frac{G2ME}{2\pi}\left[\overline{Q} + \frac{1}{3}Q\right]$$

where

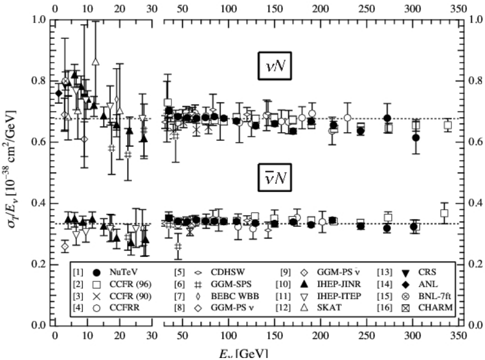
$$Q \equiv \int x[u(x) + d(x)]$$
$$\overline{Q} \equiv \int x[\overline{u}(x) + \overline{d}(x)]$$

and we have ignored the small strange component in the nucleon

- Thus

$$R_{\nu/\overline{\nu}} \equiv \frac{\sigma^{\overline{\nu}N}}{\sigma^{\nu N}} = \frac{\overline{Q} + Q/3}{Q + \overline{Q}/3} = \frac{1 + 3\overline{Q}/Q}{3 + \overline{Q}/Q}$$

Experimental Measurements of νN Scattering



- Experimentally $R_{\nu/\overline{\nu}} = 0.45 \rightarrow \overline{Q}/Q = 0.5$

There are antiquarks within the proton!

How Much Momentumn Do the Quarks and Antiquarks Carry?

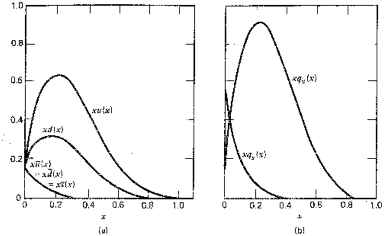


Fig. 9.9 The quark structure functions extracted from an analysis of deep inelastic scattering data. Figure (b) shows the total valence and sea quark contributions to the structure of the proton.

- Note:

$$\int xF_2^{\nu N}(x)dx = \frac{18}{5}\int xF_2^{eN}(x)dx$$

the momentum fraction that the q and $\overline{q}$ carry

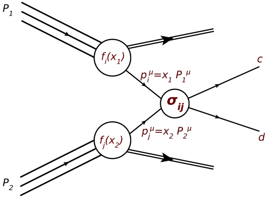
- Experimentally, SLAC found at $q^2 \sim 10$ GeV^2 that this fraction ~ 0.5

Only half the momentum of the proton is carried by quarks and a

- What's Left? The gluon!

Some Comments

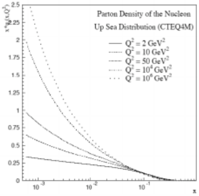
- Charged lepton probes study charged partons
- Neutrinos study all partons with weak charge
  - $\int xF_2^{\nu N}(x)dx = \frac{18}{5}\int xF_2^{eN}(x)dx$ tells us that all the weakly interacting partons are charged
- To study the gluon directly, will need a strong probe
  - No pointlike strong probes
  - Will need to convolute two pdf's

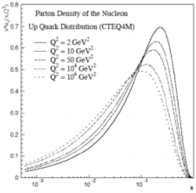


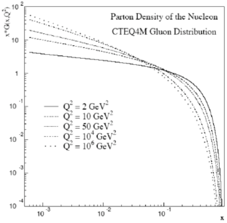
  - More on this when we talk about hadron colliders in a few weeks
- Can also indirectly study gluon by seeing how it affects the quarks

Scaling Violations in DIS

- QCD corrections to DIS come from incorporating gluon brem from the q and $\overline{q}$ and pair production $g \to q\overline{q}$
- The ability to resolve these QCD corrections are q^2 dependent
- Expected result:
  - At high x the quark pdf's decrease
  - At low x the quark and antiquark pdf's increase
- Complete treatment in QCD via coupled set of differential equations, the Alterelli-Parisi evolution equations

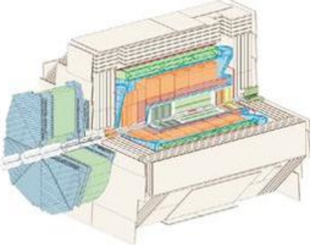




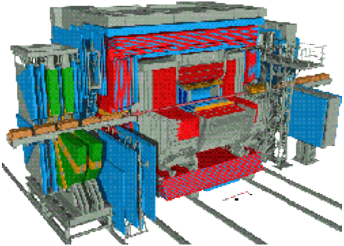


DIS in the Modern Era: The HERA collider

H1

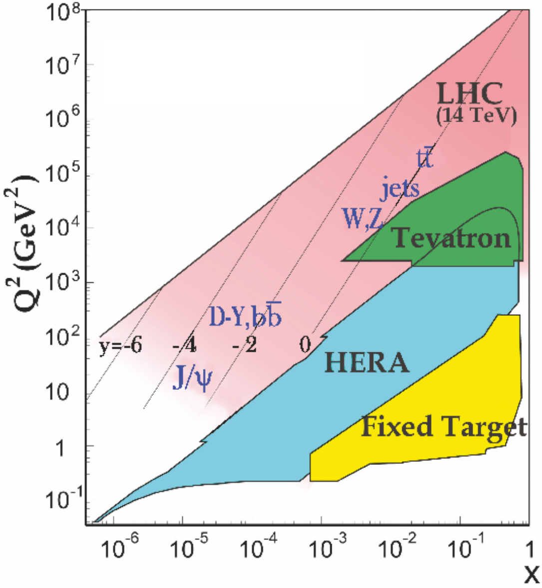


Zeus

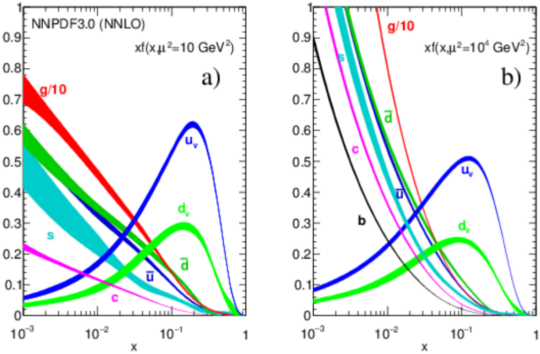


- ep collider located at DESY lab in Hamburg
- 27.5 Gev (e) x 920 GeV (p)
- Two general purpose detectors (H1 and Zeus)

What Q^2 and x are relevant?



Our best fits of PDFs at present



- Fit experimental data to theoretically motivated parameterizations
- Combine data from many experiments, using Alterelli-Parisi to account for differences in Q^2 (correct to common value)
- Analysis of uncertainties to provide a systematic uncertainty band

Modern $F_2(x, Q^2)$ Measurements